Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5559]-143

S.E. (E&TC/Elect.) (II Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** or Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagram must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve the following differential equations (any *two*) : [8]

(*i*) $\dfrac{d^3y}{dx^3}+4\dfrac{dy}{dx}=\sin 2x$

(*ii*) $\dfrac{d^2y}{dx^2}-y=\dfrac{1}{(1+e^{-x})^2}$ (By variation of parameter)

(*iii*) $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=4\cos(\log(1+x))$

(*b*) Find the Fourier transform of a function $f(x)=e^{-|x|}$. [4]

P.T.O.

Or

2. (*a*) An electric circuit consist of an inductance 'L', condenser of capacity 'C' and emf $E_0^{-\sin \omega t}$ that the charge satisfy the differential equation $\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E_0}{L}$ sin ωt. If $\omega^2 = \frac{1}{LC}$ and initially $t = 0$, $Q = 0$ and $t = 0$, find the charge at any time 't'. [4]

(*b*) Solve any *one* : [4]

(*i*) Find the z-transform of a function $f(k) = k^2 a^k$, $k \geq 0$.

(*ii*) If $f(z) = \frac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}$, then find $z^{-1}(f(z))$ for $|z| > \frac{1}{4}$.

(*c*) Solve the following difference equation $12f(k + 2) - 7f(k + 1) + f(k) = 0$; $f(0) = 0$, $f(1) = 3$, $k \geq 0$. [4]

3. (*a*) Find Lagrange's interpolating polynomial passing through set of points :

x	0	1	2
y	4	3	6

Use it to find y at $x = 1.5$; $\frac{dy}{dx}$ at $x = 0.5$. [4]

(*b*) Use Runge-Kutta method of fourth order to obtain the numerical solutions of $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$ in the interval (1, 1.1) with $h = 0.1$. [4]

(c) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [4]

Or

4. (a) Show that (any *one*) : [4]

(i) $\nabla\left(\dfrac{\bar{a}.\bar{r}}{r^5}\right) = \dfrac{\bar{a}}{r^5} - \dfrac{5(\bar{a}.\bar{r})\bar{r}}{r^7}$

(ii) $\nabla.\left[r\nabla\left(\dfrac{1}{r^3}\right)\right] = \dfrac{3}{r^4}$.

(b) If the vector field $\bar{F} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational, find a, b, c and determine ϕ such that $\bar{F} = \nabla\phi$. [4]

(c) Evaluate $\displaystyle\int_0^3 \frac{dx}{1+x}$ dividing the interval into 6 parts by using Simpson's $\dfrac{3}{8}$th rule. [4]

5. (a) Evaluate $\displaystyle\int_c \bar{F}.d\bar{r}$ for $\bar{F} = 2xy\bar{i} + (x^2 - i)\bar{j} + yz\bar{k}$ along a straight line joining $(0, 0, 0)$ and $(1, 2, 1)$. [4]

(b) Use Stokes' theorem to evaluate $\displaystyle\int_c (2y\bar{i} + z\bar{j} + 3y\bar{k}).d\bar{r}$ where c is boundry of rectangle $0 \le x \le 2$, $0 \le y \le 3$, $z = 1$. [4]

(c) By using Gauss-Divergence theorem evaluate $\displaystyle\iint_s (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}).d\bar{s}$ over the surface of sphere $x^2 + y^2 + z^2 = 1$. [5]

Or

6. (*a*) Using Green's theorem evaluate $\int_c (2 - xy)dx + y^2\, dy$ over boundary of region enclosed by parabola $y^2 = x$, line $x = 1$ and x-axis in first quadrant. [4]

(*b*) By using Stokes' theorem evaluate $\iint_s (\nabla \times \overline{F}).d\overline{s}$ where $\overline{F} = y\overline{i} + (x - 2xz)\overline{j} - xy\overline{k}$ over surface of hemisphere $x^2 + y^2 + z^2 = a^2$ over xy-plane. [5]

(*c*) By using Gauss-Divergence theorem evaluate $\iint_s (2xy\overline{i} + yz^2\overline{j} + x^2y\overline{k}).d\overline{s}$ over total surface of region bounded by $x = 0$, $y = 0$, $z = 0$, $y = 3$ and $x + 2z = 6$. [4]

7. (*a*) If $f(z) = u + iv$ is an analytic function show that : [4]

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(z)|^2 .$$

(*b*) Evaluate : [5]

$$\int_c \frac{4z^2 + z}{z^2 - 1}\, dz \quad \text{where } c \text{ is } |z - 1| = \frac{1}{2}.$$

(*c*) Find the bilinear transformation which maps the points $z = 2, 1, 0$ from z-plane onto the points $w = 1, 0, i$ of w-plane. [4]

Or

8. (*a*) If $u = 3x^2 - 3y^2 + 2y$ find v, such that the function $f(z) = u + iv$ is an analytic function. [4]

(*b*) Evaluate $\int_c \frac{4-3z}{z(z-1)(z-2)}\, dz$ where c is the contour $|z| = 3/2$. [5]

(*c*) Find image of X-axis under the transformation $w = \frac{i-z}{i+z}\cdot$ [4]