Total No. of Questions—**8**] [Total No. of Printed Pages—**4**

Seat No.	

[5459]-143

S.E. (E&TC/Elect.) (Second Semester) EXAMINATION, 2018
ENGINEERING MATHEMATICS—III
(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* **2**, Q. No. **3** *or* **4**, Q. No. **5** *or* **6**, Q. No. **7** *or* **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve the following differential equations (any *two*) : [8]

(*i*) $\dfrac{d^3y}{dx^3} + 4\dfrac{dy}{dx} = \sin 2x + 2^x$

(*ii*) $\dfrac{d^2y}{dx^2} + y = \sec x \,.\, \tan x$ (By variation of parameter)

(*iii*) $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x^5$

(*b*) Find the Fourier sine transform of a function $f(x) = e^{-|x|}$. [4]

Or

2. (*a*) An electric circuit consists of an inductance 'L', condenser of capacity 'C' and emf '$E_0 \,.\, \cos \omega t$' so that the charge Q satisfies the differential equation $\dfrac{d^2Q}{dt^2} + \dfrac{Q}{LC} = \dfrac{E_0}{L}\cos \omega t$. If $\omega^2 = \dfrac{1}{LC}$ and at $t = 0$, $Q = Q_0$ and $i = i_0$, find the charge at any time 't.' [4]

P.T.O.

(b) Solve any *one* : [4]

(i) Find z-transform of a function $f(k) = (k+1)2^k, k \geq 0$.

(ii) Find the inverse z-transform of $f(z) = \frac{z}{(z-2)(z-3)} |z| > 3$.

(c) Solve the following difference equation : [4]

$$f(k+1) + \frac{1}{2}f(k) = \left(\frac{1}{2}\right)^k, k \geq 0, f(0) = 0.$$

3. (a) Find Lagrange's interpolating polynomial passing through set of points : [4]

x	0	1	2
y	2	5	10

Use it to find y at $x = 0.5$; $\frac{dy}{dx}$ at $x = 0$.

(b) Compute $y(0.1)$ by Runge-Kutta method of 4th order for the differential equation : [4]

$$\frac{dy}{dx} = xy + y^2, y(0) = 1 \text{ with } h = 0.1$$

(c) If the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to x-axis, find the values of a, b, c. [4]

Or

4. (a) Show that (any *one*) : [4]

(i) $\nabla[\nabla . (\bar{r} / r^2)] = \frac{-2}{r^4}\bar{r}$

(ii) $\nabla[\bar{a} . \nabla \log r] = \frac{\bar{a}}{r^2} - \frac{2(\bar{a} . \bar{r})}{r^4}\bar{r}$

(*b*) Show that the vector field $\overline{F} = (x^2 - yz)\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$ is irrotational. Find scalar potential ϕ such that $\overline{F} = \nabla\phi$. [4]

(*c*) Evaluate $\int_1^2 \frac{dx}{x^2}$ using Simpson's $\frac{1}{3}$rd rule taking $h = 0.25$. [4]

5. (*a*) Find work done in moving a particle once around the circle $x^2 + y^2 = 1, z = 1$ in the force field $\overline{F} = z\overline{i} + x\overline{j} + y\overline{k}$. [4]

(*b*) Using Stokes' theorem evaluate $\int_C (y\,dx + z\,dy + x\,dz)$ where C is the boundary of rectangle $0 \le x \le 2$, $0 \le y \le \pi$, $z = 3$. [4]

(*c*) Use Gauss-Divergence theorem to evaluate $\iint_S (y^2z^2\overline{i} + x^2z^2\overline{j} + x^2y^2\overline{k}) \cdot d\overline{S}$ where S is the surface of hemisphere $x^2 + y^2 + z^2 = 9$ above xy-plane. [5]

Or

6. (*a*) Using Green's theorem evaluate $\int_C (x^3 - 2y^2)\,dx + (3xy + 4x^2)dy$ along the closed curve formed by $x = 0$, $x = 1$, $y = 0$ and $y = 2$. [4]

(*b*) By using Stokes' theorem evaluate $\iint_S (\nabla \times \overline{F}) \cdot d\overline{S}$ where $\overline{F} = y\overline{i} + z\overline{j} + x\overline{k}$ over surface of paraboloid $z = 1 - x^2 - y^2$ for which $z \ge 0$. [4]

(*c*) By Gauss-Divergence theorem evaluate $\iint_S (x^3\overline{i} + y^3\overline{j} + z^3\overline{k}) \cdot d\overline{S}$ over the surface of the sphere $x^2 + y^2 + z^2 = 1$. [5]

7. (*a*) If $f(z) = u + iv$ is an analytic function show that both u and v are harmonic. [4]

(*b*) Evaluate : [5]

$$\int_C \frac{\sin 2z}{\left(z + \frac{\pi}{3}\right)^2} dz$$

where C is the contour $|z| = 2$.

(*c*) Find the bilinear transformation which maps the points 1, i, -1 from z-plane onto the points i, 0, $-i$ of W-plane. [4]

Or

8. (*a*) Find harmonic conjugate of $u = x^3 - 3xy^2$ and corresponding analytic function in terms of z. [4]

(*b*) Evaluate $\int_C \frac{z^3 - 5}{(z+1)^2 (z-2)} dz$ where C is the contour $|z| = 3$. [5]

(*c*) Find image of straight line $y = x$ under the transformation $w = \frac{z-1}{z+1}$. [4]