Total No. of Questions—8] [Total No. of Printed Pages—5

Seat No.	

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S.E. (E&TC/(Electronics Engg.) (Second Semester) EXAMINATION, 2017

ENGINEERING MATHEMATICS–III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. **1** *or* Q. **2**, Q. **3** *or* Q. **4**, Q. **5** *or* Q. **6**, Q. **7** *or* Q. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve (any *two*) : [8]

(*i*) $(D^2 + 4D + 4)y = \sin 2x$

(*ii*) $(D^2 - 6D + 9)y = \dfrac{e^{3x}}{x^2}$

(*iii*) $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^2$

(*b*) Find $f(x)$ if $F_c(\lambda) = e^{-3\lambda}$, $\lambda > 0$. [4]

Or

2. (*a*) A capacitor 10^{-3} farad is in series with e.m.f. of 20 volts and an inductor of 0.4 henry, at $t = 0$, the charge Q and current I are zero, find Q at any time t. [4]

P.T.O.

(*b*) Find the inverse z–transform (any *one*) : [4]

(*i*) $F(z) = \frac{z}{(z-1)(z-4)}, \quad |z| > 4$

(*ii*) $F(z) = \frac{1}{(z-4)(z-3)}$.

(by inversion integral method)

(*c*) Solve the following difference equation to find $f(k)$:

$f(k + 2) + 3f(k + 1) + 2f(k) = 0;$

$f(0) = 0, \; f(1) = 2, \; k \geq 0$ [4]

3. (*a*) Using fourth order Runge-Kutta method, solve the differential equation : $\frac{dy}{dx} = \frac{1}{x+y}$ with initial condition $y(0) = 1$. Find $y(0.2)$ taking $h = 0.2$ [4]

(*b*) Find Lagrange's interpolating polynomial passing through set of points : [4]

x	2	4	5
y	6	20	30

(*c*) Find the directional derivative of $\phi = xy^3 + yz^3$ at the point (2, –1, 1) in the direction of vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [4]

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla . \left[r \nabla \frac{1}{r^3} \right] = \frac{3}{r^4}$

(*ii*) $\nabla\left(\frac{\bar{a}.\bar{r}}{r^5}\right)=\frac{\bar{a}}{r^5}-5\frac{\left(\bar{a}.\bar{r}\right)\bar{r}}{r^7}$

(*b*) Show that :

$$\overline{\mathrm{F}}=(y\sin z - \sin x)\bar{i}+(x\sin z+2yz)\bar{j}+(xy\cos z+y^2)\bar{k}$$

is irrotational. Find scalar potential ϕ such that $\bar{\mathrm{F}}=\nabla\phi$. [4]

(*c*) Stating the formula for Simpson's $\frac{1}{3}$ rd rule, evaluate $\int_1^{1.04} f(x)dx$ from the following data : [4]

x :	1	1.01	1.02	1.03	1.04
$f(x)$:	3.953	4.066	4.182	4.300	4.421

5. (*a*) Find work done by the force $\overline{\mathrm{F}}=(2y+3)\bar{i}+(xz)\bar{j}+(yz-x)\bar{k}$ in taking a particle from (0,0,0) to (3, 1, 1). [4]

(*b*) Apply Stokes' theorem to calculate $\int_c 4y\,dx+2z\,dy+6y\,dz$, where c is curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. [5]

(*c*) Show that $\overline{\mathrm{E}}=-\nabla\phi-\frac{1}{c}\frac{\partial\overline{\mathrm{A}}}{\partial t}$, $\overline{\mathrm{H}}=\nabla\times\overline{\mathrm{A}}$ are solutions of the Maxwell's equations : [4]

(*i*) $\nabla\times\overline{\mathrm{H}}=\frac{1}{\mathrm{C}}\frac{\partial\overline{\mathrm{E}}}{\partial t}$

(*ii*) $\nabla \times \overline{E} = \frac{1}{c} \frac{\partial \overline{H}}{\partial t}$, if

(1) $\nabla . \overline{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$

(2) $\nabla^2 \overline{A} = \frac{1}{c^2} \frac{\partial^2 \overline{A}}{\partial t^2}$

Or

6. (*a*) Using Green's lemma evaluate $\int_c \overline{F}.d\overline{r}$, where :

$\overline{F} = \sin z\overline{i} + \cos x\overline{j} + \sin y\overline{k}$ and c is the boundary of rectangle $0 \le x \le \pi$, $0 \le y \le 1$ and $z = 3$. [4]

(*b*) Use divergence theorem to evaluate

$\iint \left(4xz\overline{i} - y^2 \overline{j} + yz\overline{k}\right).d\overline{s}$, where s is the surface of the cube bounded by the planes

$x = 0,\ x = 2,\ y = 0,\ y = 2,\ z = 0,\ z = 2$ [5]

(*c*) Prove that $\phi_C \left(\overline{a} \times \overline{r}\right).\ d\overline{r} = \iint_S 2\overline{a}.\ d\overline{s}$, where $\overline{a}$ is a constant vector.

[4]

7. (*a*) If $f(z) = u + iv$ is analytic and $u = x^2 - y^2$ find v and then $f(z)$ in terms of z. [4]

(*b*) Evaluate $\int_C \cot z\, dz$, where 'C' is the circle $|z| = 4$. [4]

(*c*) Show that under the transformation $w = z + \frac{1}{z}$, family of circles $|z| = c$ are transformed into family of ellipses in w–plane. What is the transform if $c = 1$. [5]

Or

8. (*a*) If $f(z) = u + iv$ is analytic, show that family of curves $u = c$, $v = b$ are orthogonal. [4]

(*b*) Evaluate $\int_C \frac{\sin 2z}{\left(z + \frac{\pi}{3}\right)^4} dz$, where 'C' is circle $|z| = 2$. [4]

(*c*) Find the bilinear transformation, which maps the points $0, -1, i$ of the z–plane on to the points $2, \infty, \frac{1}{2}(5+i)$ of the w–plane, respectively. [5]