Total No. of Questions—8] [Total No. of Printed Pages—6

Seat No.	

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S.E. (E&TC/Elect.) (II Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\left(D^2 + 5D + 6\right) y = e^x$

(*ii*) $\left(D^2 + 4\right) y = \sec 2x$

(by method of variation of parameters)

(*iii*) $\dfrac{dx}{x\left(2y^4 - z^4\right)} = \dfrac{dy}{y\left(z^4 - 2x^4\right)} = \dfrac{dz}{z\left(x^4 - y^4\right)}$.

P.T.O.

(*b*) Find the Fourier cosine transform of the function : [4]

$$f(x) = \begin{cases} x^2 & \text{if} \quad 0 < x < a \\ 0 & \text{if} \quad x > a \end{cases}$$

Or

2. (*a*) An uncharged condenser of capacity C charged by applying an e.m.f. of value $\mathrm{E} \sin \frac{t}{\sqrt{\mathrm{LC}}}$ through the leads of inductance L and negligible resistance. The charge Q on the plate of condenser satisfies the differential equation : [4]

$$\frac{d^2\mathrm{Q}}{dt^2} + \frac{\mathrm{Q}}{\mathrm{LC}} = \frac{\mathrm{E}}{\mathrm{L}} \sin \frac{t}{\sqrt{\mathrm{LC}}}$$

Prove that the charge at any time t is given by :

$$\mathrm{Q} = \frac{\mathrm{EC}}{2}\left[\sin \frac{t}{\sqrt{\mathrm{LC}}} - \frac{t}{\sqrt{\mathrm{LC}}} \cos \frac{t}{\sqrt{\mathrm{LC}}}\right].$$

(*b*) Find the inverse z-transform (any *one*) : [4]

(*i*) $\mathrm{F}(z) = \dfrac{2z^2 + 3z}{(z+2)(z-4)}, \quad |z| > 4$

(*ii*) $\mathrm{F}(z) = \dfrac{10z}{(z-1)(z-2)}$

(by inversion integral method)

(*c*) Solve the following difference equation : [4]

$$f(k+2) + 3f(k+1) + 2f(k) = 0;$$

$$f(0) = 0, \; f(1) = 2, \; k \geq 0.$$

3. (*a*) Solve the differential equation $\frac{dy}{dx} = 1 + xy$, $y(0) = 2$ to get $y(0.2)$ by using Runge-Kutta method of fourth order. $(h = 0.2)$ [4]

(*b*) Find Lagrange's interpolation polynomial passing through set of points : [4]

x	y
0	2
1	3
2	12
5	147

(*c*) Find directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ towards the point $\bar{i} + \bar{j} - \bar{k}$. [4]

Or

4. (*a*) Prove any *one* of the following : [4]

(*i*) $\nabla \cdot \left[r\nabla\left(\frac{1}{r^n}\right)\right] = \frac{n(n-2)}{r^{n+1}}$

(*ii*) $\nabla^4 e^r = e^r + \frac{4}{r} e^r$.

(*b*) Show that the vector field : [4]

$$\bar{F} = \left(2xz^3 + 6y\right)\bar{i} + (6x - 2yz)\bar{j} + \left(3x^2z^2 - y^2\right)\bar{k}$$

is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$.

(*c*) Compute the value of definite integral : [4]

$$\int_0^6 \frac{1}{1+x}\, dx$$

using Simpson's $\left(\frac{3}{8}\right)$th rule, dividing the interval into 6 parts.

5. (*a*) Evaluate : [4]

$$\int_C \bar{F} \cdot d\bar{r}$$

for the field $\bar{F} = x^2\, \bar{i} + xy\, \bar{j}$ over the region R enclosed by $y = x^2$ and the line $y = x$ using Green's theorem.

(*b*) Evaluate : [4]

$$\iint_S \bar{F} \cdot \hat{n}\, dS$$

for $\bar{F} = 4xz\, \bar{i} - y^2\, \bar{j} + yz\, \bar{k}$ and S, the surface of cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$ using Divergence theorem.

(*c*) Using Stokes' theorem calculate : [5]

$$\int_C 4y\, dx + 2z\, dy + 6y\, dz,$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$.

Or

6. (*a*) Find the workdone by the force field given by : [4]

$$\bar{F} = 3x^2\,\bar{i} + (2xz - y)\,\bar{j} + z\,\bar{k}$$

along the curve $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$.

(*b*) Evaluate : [4]

$$\iint_S \bar{F} \cdot d\bar{S}$$

for $\bar{F} = 4xz\,\bar{i} - y^2\,\bar{j} + yz\,\bar{k}$ and S, the surface of the cube bounded by the planes $x = 0$, $x = 3$, $y = 0$, $y = 3$, $z = 0$, $z = 3$ by using Divergence theorem.

(*c*) Evaluate : [5]

$$\iint_S (\nabla \times \bar{F}) \cdot d\bar{S}$$

for $\bar{F} = y\,\bar{i} + z\,\bar{j} + x\,\bar{k}$, where S is the surface of the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$.

7. (*a*) If $f(z) = u + iv$ is analytic, show that $u = c$, $v = b$ are orthogonal. [4]

(*b*) Evaluate : [4]

$$\oint_C \frac{z^2 + 1}{z - 3}\, dz$$

where

(*i*) 'C' is the circle $|z - 3| = 2$

(*ii*) 'C' is the circle $|z| = \frac{3}{2}$.

(*c*) Show that under the transformation $\omega = z + \dfrac{1}{z}$ family of circles $r = c$ are mapped on to family of ellipses. What happens if $c = 1$? [5]

Or

8. (*a*) If $f(z) = u + iv$ is analytic, show that u, v are Harmonic functions. [4]

(*b*) Evaluate : [4]

$$\oint_{C} \frac{\sin 2z}{\left(z+\frac{\pi}{3}\right)^4}\,dz$$

where 'C' is the circle $|z| = \dfrac{3}{2}$.

(*c*) Find the bilinear transformation which maps the points $z = -1, 0, 1$ onto the points $\omega = 0, i, 3i$ respectively. [5]